

# MODE CONVERSION AND LEAKY-WAVE EXCITATION AT OPEN-END COUPLED MICROSTRIP DISCONTINUITIES

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**Abstract** - The method of moments (MoM) is used to study mode conversion and leaky-wave excitation at an asymmetric coupled-microstrip discontinuity. The results show that significant mode conversion can occur at such discontinuities and that fundamental leaky-wave modes can be excited strongly. Numerical issues with regard to the MoM analysis of such discontinuities are addressed as well, and it is shown that inclusion of a complete-domain basis function for the fundamental leaky mode improves numerical stability dramatically.

## I. INTRODUCTION

Most of the previous studies of leaky modes supported by planar transmission lines have been two dimensional in nature, with an emphasis on calculating the complex wavenumber and the transverse field profile [1]. For the first time, this paper addresses the effects of leaky fundamental mode excitation at transmission-line discontinuities.

## II. ANALYSIS

We apply a spectral-domain Method of Moment (MoM) analysis [2] in which only the longitudinal component of current is considered since the transverse component is negligible for the line widths and frequencies of interest here. The longitudinal current density is expanded in two types of basis functions [2]: (1) semi-infinite complete-domain basis functions representing the single incident mode and the multiple reflected fundamental modes, and (2) piece-wise sinusoidal (PWS) subsectional basis functions in the vicinity of the discontinuity.

Because the excitation of fundamental leaky transmission-line modes is an important new aspect of

the present paper, we demonstrate how such modes are modeled by semi-infinite basis functions. Assuming the modal wavenumber  $\gamma = j\beta + \alpha$  ( $\beta > 0$  and  $\alpha > 0$ ), the basis function's longitudinal variation for propagation in the  $-z$  direction is expressed as

$$\exp[j(\beta + \alpha)z]U(-z) = \exp(\alpha z)[\cos(\beta z) + (1) \quad j\sin(\beta z)]U(-z)$$

where as discussed in [2] it is convenient to represent the basis function in terms of real trigonometric functions. For an arbitrary function  $g(z)$  we define the transform pair

$$g(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{g}(k_z) \exp(-jk_z z) dk_z \leftrightarrow \quad (2) \quad \hat{g}(k_z) = \int_{-\infty}^{\infty} g(z) \exp(jk_z z) dz$$

and from simple residue calculus

$$\exp(\alpha z) \cos(\beta z) U(-z) \leftrightarrow \quad (3) \quad \frac{1}{2j} \left[ \frac{1}{k_z + \beta - j\alpha} + \frac{1}{k_z - \beta - j\alpha} \right]$$

where the integration path in the  $k_z$  plane is along the real axis, with a similar result for  $j\exp(\alpha z)\sin(\beta z)U(-z)$ . For nonleaky modes ( $\alpha=0$ ) the semi-infinite sinusoidal basis functions introduce poles along the real  $k_z$  axis, and for leaky modes these poles become complex and reside in the upper half of the complex plane. In the evaluation of the spectral-domain integrals required for the elements of the MoM matrix, the integral in the immediate vicinity of real-axis poles (associated with surface waves and nonleaky transmission-line-mode basis functions) is evaluated using residue calculus, while the remainder of the integral is evaluated via Gauss-Legendre

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numerical integration.

### III. RESULTS

We consider the following operating conditions (see Fig. 1): frequency  $f=2.55$  GHz ( $\lambda=11.76$  cm),  $w/\lambda=0.034$ ,  $s/\lambda=0.068$ ,  $d/\lambda=0.034$ , and  $\epsilon_r = 2.55$ .

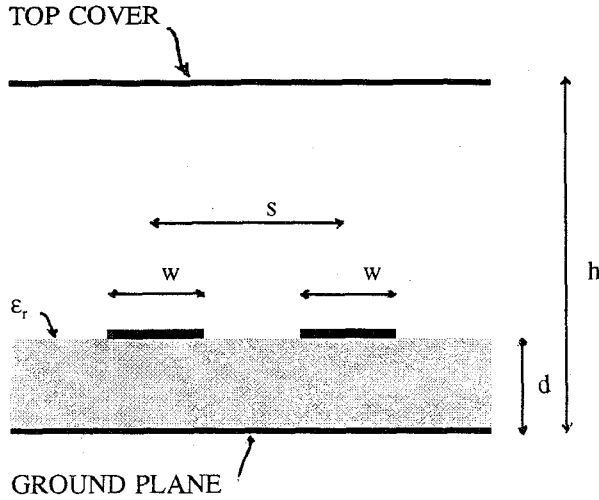


Figure 1. Cross-sectional view of a coupled-microstrip transmission line with a top cover. The top cover, ground plane, and dielectric substrate are assumed in the analysis to be of infinite transverse extent.

#### A. Mode Conversion

Our first example corresponds to a situation in which the top cover is distant from the strips ( $h=5d$ ) and therefore provides minimal perturbation to the fields in the strip vicinity. Thus, although there are  $N=4$  conductors, there are only two zero-cutoff-frequency modes of importance: the even and odd coupled-microstrip modes, each of which is represented by a semi-infinite basis function. The third fundamental mode, corresponding to the parallel-plate mode weakly perturbed by the presence of the strips, is not represented explicitly in the current expansion (the justification for excluding this mode in the complete-domain basis function expansion is discussed further in Sec. IIIC).

The even coupled-microstrip mode (mode E1) is assumed incident on the open-circuit discontinuity and we examine the relative excitation

strengths of the even and odd (mode O) modes. One of the strips is abruptly terminated in an open circuit at  $z = 0$ , while the other is terminated in an open circuit at  $z = +L$ , for various values of  $L$ . The reflected modes travel in the  $-z$  direction. The first step in the numerical procedure is to solve a two-dimensional problem for the modal wavenumbers, which for the nonleaky case considered here are  $\beta_{\text{even}}/k_0=1.47$  and  $\beta_{\text{odd}}/k_0=1.37$ . The excitation strengths of modes E1 and O are shown in Fig. 2 as a function of the length extension  $L/\lambda_g$ , where  $\lambda_g$  is the guide wavelength of an isolated microstrip line of width  $w$  in the same inhomogeneous parallel-plate waveguide, and  $L$  is the difference in length of the two strips.

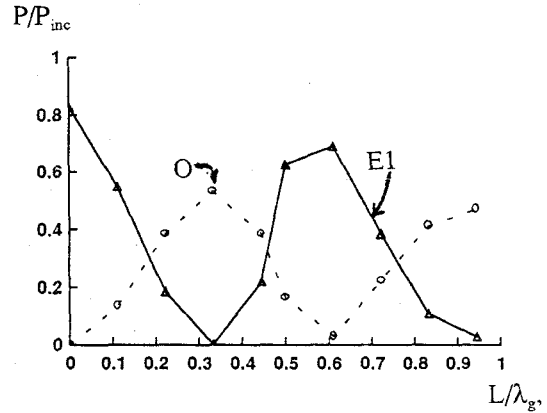


Figure 2. Relative excitation power  $P/P_{\text{inc}}$  of the even (E1) and odd (O) coupled-microstrip modes due to the incidence of mode E1 (incident power  $P_{\text{inc}}$ ) upon the discontinuity in Fig. 1.

From simple transmission-line theory, we anticipate that for  $L/\lambda_g=0.25$  the input impedance at  $z=0$  for the extended strip is approximately that of a short circuit while the input impedance on the other strip is close to an open. Thus, for this length extension the reflected current density on the two strips should be of nearly opposite polarity and thus the odd mode should be excited strongly; the same arguments apply for  $L/\lambda_g \approx 0.75$ . However, when  $L=0$  the symmetry of the discontinuity precludes excitation of the odd mode; this also leads to the anticipation that the odd mode will be excited weakly for  $L/\lambda_g \approx 0.5$ . This simple theory explains the oscillatory numerical results in Fig. 2. However, the situation in the actual scattering problem is much more complicated than in the simplified transmission-line model and therefore the lengths  $L/\lambda_g$  computed in the

numerical calculations are shifted slightly from the values anticipated from the simplified theory.

### B. Leaky-Wave Excitation

We now consider cover heights in the range  $h/\lambda=0.042$  to  $h/\lambda=0.068$ , with corresponding wavenumbers plotted in Fig. 3; this structure supports one odd and two even fundamental modes [1]. For the cover heights considered here one of the even modes (mode E1) is never leaky, the other even mode (mode E2) is always leaky, and the odd mode (mode O) is leaky for some cover heights and nonleaky for others. Results of calculations not shown here indicate that modes E1 and O are respectively the even and odd coupled-microstrip modes perturbed by the presence of the top cover, and mode E2 is the parallel-plate mode perturbed by the presence of the strips. As in Sec. IIIA, mode E1 is assumed incident upon the discontinuity, and we investigate the excitation strengths of all three fundamental modes.

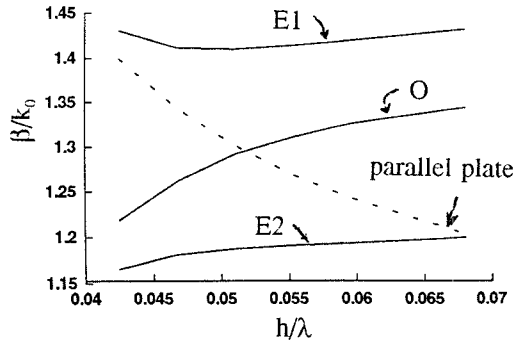


Figure 3. Wavenumbers for the three zero-cutoff-frequency modes supported by the structure in Fig. 1. Also shown (dashed) is the wavenumber of the parallel-plate mode.

Results are shown in Fig. 4 for the case in which the cover height is  $h/d=1.625$  and the amplitude of mode O is normalized by the factor  $(Z_e/Z_o)^{1/2}$ , where  $Z_e=102\Omega$  and  $Z_o=75\Omega$  are the characteristic impedances [3] of modes E1 and O, respectively; we do not normalize the amplitude of mode E2 since it is leaky and a meaningful

characteristic impedance cannot be defined for this mode. However, the current densities for modes E1 and E2 have similar form and therefore their relative amplitudes do reflect meaningfully on their relative excitation strengths. As in Fig. 2, the odd mode is not excited for  $L=0$  and is weakly excited for  $L/\lambda_g \approx 0.6$ , with strong excitation for  $L/\lambda_g \approx 0.35$ . However, for values of  $L$  for which E1 was excited strongly in Fig. 2, both modes E1 and E2 are excited strongly.

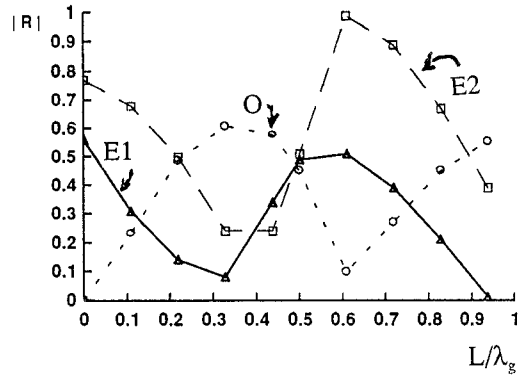
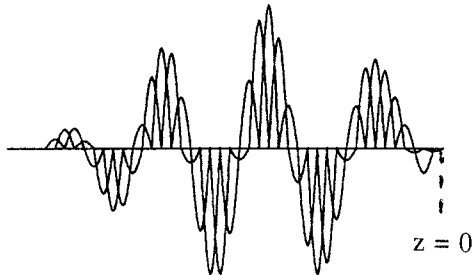


Figure 4. Magnitude of modes E1, E2 and O excited at the asymmetric coupled-microstrip discontinuity in Fig. 1. The geometrical parameters are as in Fig. 2, except that  $h/d=1.625$ .

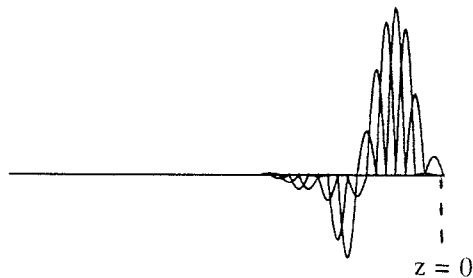
### C. Importance of Mode E2 and Numerical-Stability Issues

To examine the importance of expanding mode E2 in a complete-domain basis, the example in Fig. 4 was repeated but complete-domain basis functions were included only for modes E1 and O. With this current expansion a convergent and stable solution proved to be difficult. It was found that the results started to converge when the PWS functions were extended several wavelengths away from the discontinuity. However, the results tended to be unstable, especially the results for E1. We found, however, that if the PWSs are extended far enough away from the discontinuity, they attempt to model the currents of the leaky mode E2 which is not modeled with a complete-domain basis function (see Fig. 5). The pattern that they trace out is seen to correspond approximately to a decaying mode, with a wavelength that is close to that of mode E2. Once a complete-domain expansion function for E2 is included in the calculations, the results become very

stable, and convergence of the PWS occurs more rapidly (see Fig. 6).



**Figure 5.** Piece-wise sinusoidal (PWS) basis functions used to model the currents excited by the discontinuity in Fig. 4 for the case  $L=0$ . Complete-domain, semi-infinite basis functions were used for the incident mode E1 and for the reflected mode E1, but not for mode E2. The PWS -- weighted by their MoM-computed excitation strength -- appear to simulate the damped (leaky) mode E2.



**Figure 6.** Piece-wise sinusoidal (PWS) basis functions used to model the currents excited by the discontinuity in Fig. 4 for the case  $L=0$ . The results are plotted as in Fig. 5, but in this case a semi-infinite basis function was used to model mode E2 (as was done in the results in Fig. 4).

Returning to Fig. 2, the above discussion demonstrates that the accuracy of those results does not hinge irreversibly upon the assumption that a complete-domain basis function can be excluded for mode E2 (as it was for that example). If this assumption were invalid, the PWS basis functions would attempt to model the currents associated with the excluded mode and, based on our experience, the numerical results would tend to be unstable. However, the high level of numerical stability we witnessed in computing the results in Fig. 2 -- along with the fact that for that example the PWS modes decayed quickly away from the discontinuity -- gives us confidence that our original assumption with regard to the exclusion of a complete-domain basis function for mode E2 was appropriate.

## References

- [1] L. Carin and N.K. Das, "Leaky waves on broadside-coupled microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 58-66, Jan. 1992; this reference contains a fairly complete set of references into leaky modes which, to save space, are not repeated here.
- [2] R.W. Jackson and D.M. Pozar, "Full-wave analysis of microstrip open-end and gap discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. 33, pp. 1036-1042, Oct. 1985.
- [3] L. Carin and K. J. Webb, "Characteristic impedance of multi-level, multiconductor hybrid mode microstrip," *IEEE Trans. on Magnetics.*, vol. 25, pp. 2947-2949, July 1989.